Exercise 14

Find dy/dx by implicit differentiation.

$$e^y \sin x = x + xy$$

Solution

Differentiate both sides with respect to x.

$$\frac{d}{dx}(e^y \sin x) = \frac{d}{dx}(x + xy)$$

$$\left[\frac{d}{dx}(e^y)\right] \sin x + e^y \left[\frac{d}{dx}(\sin x)\right] = \frac{d}{dx}(x) + \frac{d}{dx}(xy)$$

$$\left[(e^y) \cdot \frac{d}{dx}(y)\right] \sin x + e^y(\cos x) = \frac{d}{dx}(x) + \left[\frac{d}{dx}(x)\right] y + x \left[\frac{d}{dx}(y)\right]$$

$$(e^y y') \sin x + e^y \cos x = 1 + (1)y + x(y')$$

$$y' e^y \sin x + e^y \cos x = 1 + y + xy'$$

Solve for y'.

$$(e^y \sin x - x)y' = 1 + y - e^y \cos x$$

$$y' = \frac{1 + y - e^y \cos x}{e^y \sin x - x}$$